Aspects and Applications of Patched Grid Calculations

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Abstract

PATCHED grid calculations within the framework of an implicit, flux vector split upwind/relaxation algorithm for the Euler equations are presented. Aspects of computing on patched grids are discussed including the effect of a metric-discontinuous interface on the convergence rate of the algorithm, and the effect of curvature along an interface. Applications to a converging-diverging nozzle including effects of choking and bypass slots in two dimensions are presented.

Contents

Governing Equations and Solution Methodology

The vector form of the time-dependent Euler equations can be written in nondimensional, conservation law form as

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = 0 \tag{1}$$

where Q is the vector of conserved variables. It is standard practice to transform the equations to generalized coordinates of the form

$$\xi = \xi(x,y), \qquad \eta = \eta(x,y)$$

Even though this coordinate transformation is employed, the algorithm is constructed according to a finite-volume formulation, which allows one to maintain a freestream flow without any special consideration. The convective and pressure terms are upwind differenced with a flux vector splitting (FVS) approach, which is implemented by splitting the inviscid fluxes F and G into $F^{\pm}$ and $G^{\pm}$ according to the eigenvalues of the Jacobian matrices $\partial F/\partial Q$ and $\partial G/\partial Q$. The particular FVS technique used here was developed by Van Leer.[1] The spatial derivatives are written as flux balances across a cell; for example,

$$\frac{\partial F}{\partial x} = \frac{\partial(F^{+}F^{-})}{\partial x} = \frac{1}{\Delta x}\{[F^{+}(Q^{-}) + F^{-}(Q^{+})]_{j+\frac{1}{2}} - [F^{+}(Q^{-}) + F^{-}(Q^{+})]_{j-\frac{1}{2}}\} \tag{2}$$

The Monotonic Upstream-Centered Scheme for Conservation Laws (MUSCL)[2] approach implied by the notation $F^{+}(Q^{-})$ indicates that upwind interpolations of the dependent variable Q are being used to evaluate the split flux contributions on a cell face.

Interface Treatment

For a conservative interpolation process, the discrete form of the line integral of the flux crossing a zonal boundary must be equal to a constant with any set of points S_k used to discretize the path S. This constraint has been discussed by Rai[3,4] and can be written as

$$\sum_{k^i=1}^{K^i-1} \int_{S^i_{k-\frac{1}{2}}}^{S^i_{k+\frac{1}{2}}} F^i_k \, \mathrm{d}S = \text{constant}; \; i = 1,2, \ldots \tag{3}$$

where F^i_k is the flux evaluated on the interface of the kth cell in zone i. This can be interpreted geometrically as a requirement of equal total area under the curve of F^i vs S along the zonal boundary common to the two grids.

Consider constructing a distribution of fluxes F^2_k with points S^2_k given a known distribution F^1_k with points S^1_k on the zonal boundary. Local flux conservation yields

$$\int_{S^2_{k-\frac{1}{2}}}^{S^2_{k+\frac{1}{2}}} F^2_k \, \mathrm{d}S = \int_{S^2_{k-\frac{1}{2}}}^{S^2_{k+\frac{1}{2}}} F^1_k \, \mathrm{d}S \tag{4}$$

The right side can be written as a sum over the number of cells in zone 1 that have a common surface with the kth cell in zone 2, i.e.,

$$\int_{S^2_{k-\frac{1}{2}}}^{S^2_{k+\frac{1}{2}}} F^2_k \, \mathrm{d}S = \sum_{k^1=p}^{q} \int_{\max\left[S^1_{k-\frac{1}{2}},\, S^2_{k-\frac{1}{2}}\right]}^{\min\left[S^1_{k+\frac{1}{2}},\, S^2_{k+\frac{1}{2}}\right]} F^1_k \, \mathrm{d}S \tag{5}$$

The limits of integration at the endpoints ($k^1 = p,q$) account for the possibility that a cell in zone 1 could span one or both of the cell edges in zone 2.

In the present approach, the positive parts of the flux are computed from the upstream zone and the negative parts of the flux from the downstream zone. Thus, the negative flux on the upstream zone is interpolated from the downstream zone, and vice versa

$$\left[\int_S F^{+} \, \mathrm{d}S\right]_1 + \underbrace{\left[\int_S F^{-} \, \mathrm{d}S\right]_1}_{I} = \underbrace{\left[\int_S F^{+} \, \mathrm{d}S\right]_2}_{I} + \left[\int_S F^{-} \, \mathrm{d}S\right]_2$$

The underlined integrals are terms that require interpolation from another zone. The arrows with an "I" indicate where this information is to be obtained. This is a natural setting when using fully upwind differencing for incorporating space-marching techniques, such as supersonic Euler and parabolized Navier-Stokes, where the negative part of the flux is zero.

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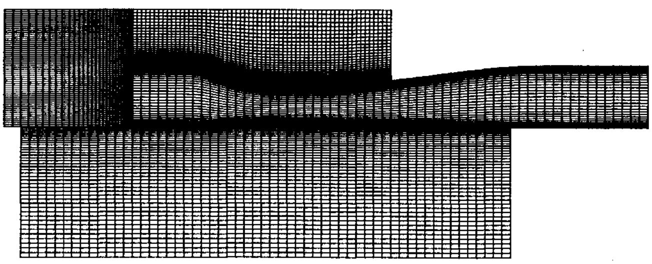
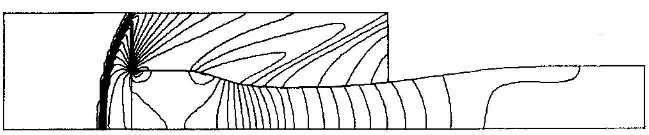
To be consistent with an implicit algorithm, terms associated with the linearization of the split-flux vectors are required on zonal boundaries. The implicit relaxation approach currently being used is the delta-form version of vertical line Gauss-Seidel relaxation with nonlinear residual updating. By updating the residual nonlinearly upon crossing zonal boundaries, the implicit terms associated with corrections across the zonal boundary are easily incorporated. An example illustrating a significant benefit in the convergence rate with implicit rather than explicit treatment at zonal interfaces is given in Ref. 5.

Another issue that warrants some discussion is the problem with curvature along an interface. The problem is the construction of an interface formulation that is both flux conserving and freestream preserving. This issue has received some attention in Ref. 3, in which an error vector is developed that measures the drift from freestream conditions as a result of applying a flux-conserving interface formulation. It is shown that the error is related to the curvature of the interface and, in general, is small in magnitude.

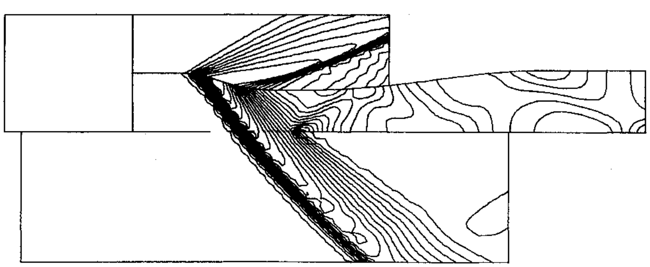
The geometric interpretation of the problem is simple: errors are introduced by the voids and/or overlaps of the two grids at an interface. One drastic solution is to simply allow no curvature on the interface, which limits the generality of the patched grid approach. Another solution, which ensures that the interface geometry be contiguous between the two zones, is to share a fixed number of the mesh points on the interface between both zones when the curvature is nonzero. This guarantees that there will be no voids or overlapping of the grids along the interface and does not result in a very stringent constraint on patching grids together. In three dimensions, the problem is considerably more difficult and it may not be possible or desirable to avoid curvature at an interface.



a) Four zone grid.



b) Pressure contours indicating choked flow.



c) Pressure contours sliding door open.

Fig. 1 Multiblock inlet

Results

As a test of the interface formulation, the flow through a converging-diverging duct is considered. A typical inlet and its surrounding grids are shown in Fig. 1a. For this configuration, it is known that the Mach number at which aerodynamic choking begins is 1.9.[6] In this case, a shock wave initially forms within the nozzle and begins to travel upstream to the front of the inlet. If only the single grid is used, the convergence rate of the algorithm will stall because the physics cannot be represented within the grid contained in the nozzle (see Ref. 5). On the other hand, if the composite grid is used, the shock wave can "pop out" of the inlet forming a bow shock and allowing the appropriate amount of mass to spill over the edge of the inlet as shown in Fig. 1b.

One way of operating the nozzle at $M_\infty = 1.9$ or lower without choking the flow is by adding a sliding door on the bottom wall of the inlet. The door can be opened to allow the shock to pass through the door and mass to spill out of the inlet. The patched grid approach easily accommodates this type of simulation by adding another grid underneath the inlet. A calculation with an open door was performed at $M_\infty = 1.9$. Pressure contours from a third-order accurate solution are shown in Fig. 1c for this case.

Conclusions

Patched grid calculations along with other techniques such as local mesh refinement will play an increasingly important role in computational fluid dynamics calculations, particularly in three dimensions. Although no one scheme or approach is completely satisfactory for complex geometries, it appears that the level of sophistication of the patched grid approach in conjunction with a shock-capturing algorithm is sufficient to use as a preliminary design tool.

References

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